Name:_____ Date:_____

Calculus Review Quarter 4 Test 2 Mr. Callahan

This is some review. Be sure to study your class notes and examples as well!!

Topics:

- Area Between Curves
- Length of an Arc

1. Find the area between $f(x) = x^2 - 1$ and the x-axis on the interval [0,2]
2. Find the area bound by $f(x) = \sin x$ and $g(x) = x^2$
3. Find the length of the curve given by the function $y = 2x^{3/2} - 1$ on the interval [0,1].
4. What is the length of $g(x) = x^{3/2}$ on the interval [0,1]?
5. Find the length of $y = \frac{2}{3}\left(x^2 + 1\right)^{3/2}$ from $x = 1$ to $x = 4$.
6. What is the area between $f(x) = 2x^2 - 8x + 10$ and $g(x) = \frac{1}{2}x^2 - 2x - 1$ on the interval [1,3].
7. Find the area bounded by $y = \frac{-x^3}{2} + 2x^2$ and $y = -x^2 + 4x$.
8. What is the area bounded by $f(x) = \frac{x^2}{2} - 3x - \frac{1}{2}$ and $g(x) = 3$?

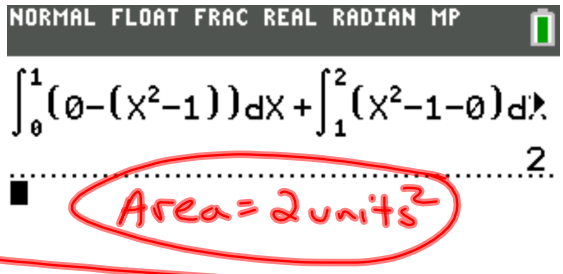
1) Find where x^2-1 crosses x-axis

$x^2-1=0$

$(x+1)(x-1)=0$

$x=-1 \quad x=1$

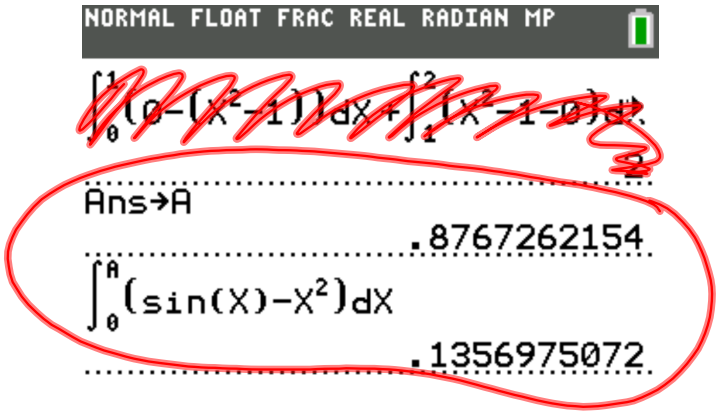
$$\text{Area}=\int_0^1 (0-(x^2-1))\,dx+\int_1^2 (x^2-1-0)\,dx$$



2) $\sin(x)$ and x^2

They cross @ $x=0$ and $x=.877$

$$\text{Area}=\int_0^{.877} (\sin(x)-x^2)\,dx = .136 \text{ units}^2$$



3) $y = 2x^{3/2} - 1$

$y' = 3x^{\frac{1}{2}}$

$\text{Length} = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$

$\text{Length} = \int_0^1 \sqrt{1 + (3x^{1/2})^2}\, dx$

NORMAL FLOAT FRAC REAL RADIAN MP

$\int_0^1 \left(\sqrt{1+\left(3X^{\frac{1}{2}}\right)^2}\right) dX$

2.268353822

Length = 2.268 units

4) $g(x) = x^{3/2}$

$g'(x) = \frac{3}{2}x^{1/2}$

$\text{Length} = \int_0^1 \sqrt{1 + \left(\frac{3}{2}x^{1/2}\right)^2}\, dx$

NORMAL FLOAT FRAC REAL RADIAN MP

$\int_0^1 \left(\sqrt{1+\left(\frac{3}{2}X^{1/2}\right)^2}\right) dX$

1.439709873

Length = 1.440 units

5) $y = \frac{2}{3}(x^2+1)^{3/2}$

$y' = 1(x^2+1)^{1/2}(2x)$

$\text{Length} = \int_1^4 \sqrt{1 + \left((x^2+1)^{1/2}(2x)\right)^2}\, dx$

NORMAL FLOAT FRAC REAL RADIAN MP

$\int_1^4 \left(\sqrt{1+\left((X^2+1)^{1/2}(2X)\right)^2}\right) dX$

45 units

6) $f(x) = 2x^2 - 8x + 10$ ← Top

$g(x) = \frac{1}{2}x^2 - 2x - 1$ ← Bottom

$[1,3]$

$\text{Area} = \int_1^3 \left(2x^2 - 8x + 10 - \left(\frac{1}{2}x^2 - 2x - 1\right)\right) dx$

NORMAL FLOAT FRAC REAL RADIAN MP

$\int_1^3 \left(2X^2 - 8X + 10 - \left(\frac{1}{2}X^2 - 2X - 1\right)\right) dX$

11 units^2

7) $y = \frac{-x^3}{2} + 2x^2$ $\quad y = -x^2 + 4x$

$2\left(\frac{-x^3}{2} + 2x^2 = -x^2 + 4x\right)$

$-x^3 + 4x^2 = -2x^2 + 8x$

$+2x^2 - 8x \quad +2x^2 - 8x$

$-x^3 + 6x^2 - 8x = 0$

$-x(x^2 - 6x + 8) = 0$

$-x(x-4)(x-2) = 0$

$x = 0 \mid x = 4 \mid x = 2$

$$\text{Area} = \int_0^2 \left(-x^2 + 4x - \left(\frac{-x^3}{2} + 2x^2\right)\right) dx$$

$$+ \int_2^4 \left(\frac{-x^3}{2} + 2x^2 - \left(-x^2 + 4x\right)\right) dx$$

4 units^2

8) $f(x) = \frac{x^2}{2} - 3x - \frac{1}{2}$ $g(x) = 3$

$$2\left(\frac{x^2}{2} - 3x - \frac{1}{2} = 3\right)$$

$$x^2 - 6x - 1 = 6$$

$$x^2 - 6x - 7 = 0$$

$$(x-7)(x+1) = 0$$

$x = 7$ $x = -1$

$$\text{Area} = \int_{-1}^{7} \left(3 - \left(\frac{x^2}{2} - 3x - \frac{1}{2}\right)\right) dx$$

$$\text{Area} = \frac{128}{3} \text{ units}^2$$